Three-Phase Induction Generator Feeding a Single-Phase Electrical Distribution System
- Time Domain Mathematical Model

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ABSTRACT

This paper presents a time domain mathematical model of a three phase induction generator feeding a single-phase electrical distribution system. The objective in this work is to analyze the three-phase induction generator when working in the conditions above described. From such model it is possible to make a more precise analysis, since some problems should be easily observed using the time domain technique, such as unbalancing of magnetomotive forces (mmf) at the machine inner magnetic circuit. This is impossible to be visualized when using frequency domain modeling. A 2 HP three-phase induction generator model is used on digital simulation. Through the results it is easy to show that the proposed model is very efficient.

Keywords: induction generator, single-phase, distribution system

1 INTRODUCTION

Countries with a large territorial extension, where electrical energy consumers are characterized by:
- small monthly kWh expenditure;
- small number of consumers per km of distribution network;
- small maximum simultaneous demands;
- and also a small amount of financial resources to be spent in rural electrification programs.

Having the Brazilian scenery as our example, the electrical energy authorities turned to single-phase (one wire with earth return) as a less expensive option to electrical distribution systems in rural areas. In the production side, due to a constant pressure of the technology to become more efficient and to increase the maximum demand, the rural consumer becomes locked to the limitations of the single-phase system.

However, in some regions where small hydroelectric potentials are available, there's a possibility to generate energy with the use of three-phase induction machines, with squirrel cage rotor connected to a single-phase distribution system in order to supply typical rural three-phase loads.

Some studies have already been done about the three-phase induction generator connected to a single-phase energy system. Those studies were of great importance but developed through frequency domain mathematical models[1]. The use of frequency domain techniques doesn't allow the analysis for the behavior of the three-phase induction generator concerning the electromagnetic unbalances inside the machine. So, aware of this matter this paper presents a time domain mathematical model [2][3][4], which allows such studies to be made. These studies show the electromagnetic behavior in the three-phase induction generator inner electromagnetic circuit.

2 INDUCTION GENERATOR

The proposed three-phase induction generator is connected to a single phase electrical distribution system. It is an ordinary induction machine with a squirrel cage rotor and stator phases displaced spatially by 120° and same number turns/phase. An adequate capacitance $C_p$ is coupled between phases $B$ and $C$, as in Figure 1. The objective is to obtain a reliable operating point for this unbalanced configuration, where the generator is delivering nominal power under nominal voltage. As described in reference [1], a balanced situation between generator phases can be obtained, and this balance is of fundamental importance concerning the connection of a three-phase induction generator in parallel with a single-phase voltage source. Through the frequency domain mathematical modeling proposed in [1], it is clear the good behavior for the proposed generator, in terms of "rms" values, where the voltage unbalance is not so accentuated. However, an interesting point not brought in picture in [1] is the behavior of electromagnetic torque inside the generator, for small voltage unbalancing the internal electromagnetic torque shows a swinging behavior. Those swings are related to the current unbalance in the generator phases as part of a unbalanced three-phase system. As a consequence this generator has different magnetomotive forces (mmf) in each phase. This unbalancing allows the decomposition of the mmf distribution in such a way to obtain static and rotary magnetic field distribution, where the latter are of positive and negative sequence. The positive sequence magnetic field, $B_{R1}$, under the three-phase symmetrical induction machine working principles, create indirectly a rotating magnetic field, $B_{S1}$, of same sequence, in the machine's rotor. The same happens to the negative sequence magnetic field, $B_{S2}$, creating another $B_{R2}$. The above is illustrated in Figure 2, and can be seen clearly that:

- The magnetic fields ($B_{S1}$, $B_{R1}$) and ($B_{S2}$, $B_{R2}$) show a constant angular displacement between them, therefore the resulting aligning electromagnetic torque is constant;
The angular position between magnetic fields (B_S1, B_R1) and (B_S2, B_R2) changes periodically, therefore the electromagnetic torque is an oscillating value.

Figure 1 – Connection diagram for the three-phase squirrel-cage induction generator. Connected to a single-phase distribution system.

Figure 2 – Demonstration of the internal rotating magnetic fields inside a three-phase induction generator connected to a single-phase system.

Frequency domain mathematical models doesn’t allow instantaneous analysis, therefore an induction generator time domain mathematical model was built in order to do this research.

3 MATHEMATICAL MODEL

Figure 1 shows the connection diagram for the three-phase induction generator connected to a single-phase distribution system.

From Figure 1 we have:

\[ \frac{d(V_b - V_c)}{dt} = \frac{1}{Cap} L_{Cap} \]

(1)

\[ i_e = -(i_a + i_b) \]

(2)

\[ V = V_a - V_c \]

(3)

\[ V_a - V_b = R_i_{ab} \]

(4)

\[ V_b - V_c = R_i_{bc} \]

(5)

\[ V_c - V_a = R_i_{ca} \]

(6)

The induction generator voltage and current relationship is presented by the generic equation:

\[ v_i = r_i i_i + \frac{\partial \lambda_i}{dt} \]

(7)

where: \( v_i, i_i, r_i, \lambda_i \) – are respectively voltage, current, resistance and coupled magnetic flux for one of the generator’s phase called. \( i \) index – represents one of the generator’s phase (stator abc – rotor ABC).

The magnetic flux coupling are given by the following general equation:

\[ \lambda_i = (L_i + L_{ij})i + \sum_{i,j} L_{ij}i \]

(8)

where:

\( L_i \) – generator’s phase i self inductance,

\( L_{ij} \) – generator’s mutual inductance between phases i and j,

\( i_i, i_j \) – instantaneous currents of phases i and j, respectively.

For the three-phase generator pictured in Figure 1, self and mutual inductances are given by [L] matrix as follows:

\[
[L] = k \begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\
A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\
A_{31} & A_{32} & A_{33} & 0 & 0 \\
A_{41} & A_{42} & 0 & A_{33} & 0 \\
A_{51} & A_{52} & 0 & 0 & A_{33}
\end{bmatrix}
\]

(9)

The [L] matrix terms are given by:

\[
A_{ij} = L_{ij} (1 - \cos \left( \frac{2 \pi}{3} \right)) + K_L
\]

\[
A_{11} = L_{11} - 2L_{13} + L_{33}
\]

\[
A_{12} = L_{12} - L_{23} - L_{13} + L_{33}
\]

\[
A_{13} = L_{14} - L_{34}
\]

\[
A_{14} = L_{43} - L_{35}
\]

\[
A_{15} = L_{46} - L_{36}
\]

\[
A_{22} = L_{22} - 2L_{23} + L_{33}
\]

\[
A_{23} = L_{24} - L_{34}
\]

\[
A_{24} = L_{25} - L_{35}
\]

\[
A_{25} = L_{26} - L_{36}
\]

\[
A_{33} = L_{44}
\]

The \( L_{ij} \) values are given below.

\[
L_{41} = 1 - \cos \left( \frac{2 \pi}{3} \right)
\]

\[
L_{42} = L_{43} = L_{23} = \cos \left( \frac{2 \pi}{3} \right)
\]

\[
L_{44} = L_{25} = L_{36} = \cos (\theta)
\]

\[
L_{45} = L_{26} = L_{43} = \cos \left( \theta + \frac{2 \pi}{3} \right)
\]

\[
L_{46} = L_{24} = L_{35} = \cos \left( \theta - \frac{2 \pi}{3} \right)
\]

K_L – is the machine magnetic circuit constant.

Introducing the three-phase induction generator mechanical equations, they are given by:
\[
\frac{dW_R}{dt} = \frac{1}{J} (T_p - T_m) \tag{10}
\]
\[
W_R = \frac{d\theta_R}{dt} \tag{11}
\]
\[
\frac{d\theta}{dt} = \frac{p}{2} \omega_R \tag{12}
\]
\[
T_m = \frac{p}{4} [I]^T \left[ \frac{\partial L}{\partial I} \right] [I] \tag{13}
\]

where:
- \( W_R \) – rotor angular mechanical speed;
- \( \theta \) - rotor electrical angular displacement;
- \( p \) – pole number;
- \( T_m \) – generator electromagnetic torque;
- \( T_b \) - turbine torque.

From the union of electrical (1) to (9) and mechanical (10) to (13) equations, can be obtained an equation system which represents the three-phase induction generator connected to a single-phase distribution system. The resulting matrix equation system is shown below:

\[
\frac{d[I]}{dt} = \left[ L' \right]^{-1} \left[ [V'] - \left[ R' \right] + \left[ \frac{d[L']}{dt} \right] [I] \right]
\]

where:
- \([I']\) - represents the current matrix (stator, rotor and load);
- \([L']\) - represents the inductance matrix (stator, rotor and load);
- \([V']\) - represents the voltage matrix (stator, rotor and load);
- \([R']\) – represents the resistance matrix (stator, rotor and load).

**Table 1.** Three-phase induction machine equivalent circuit parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator Resistance</td>
<td>3.80±0.03Ω</td>
</tr>
<tr>
<td>Rotor Resistance (referred to the stator)</td>
<td>3.01±0.03Ω</td>
</tr>
<tr>
<td>Locked Rotor Reactance ( referred to the stator)</td>
<td>3.10±0.03Ω</td>
</tr>
<tr>
<td>Phase Magnetization Reactance</td>
<td>75.15±0.7Ω</td>
</tr>
</tbody>
</table>

**4 DIGITAL SIMULATION**

Once having all the induction generator equations, digital simulations have been done in order to confirm the electromagnetic torque oscillations. Therefore, the benchmark was the three-phase induction motor used in [1], a three-phase induction machine with similar characteristics was taken, a 2 HP, 4 poles, 380/220 Volts, squirrel cage rotor, with equivalent circuit parameters given by Table 1.

The following simulations are obtained with respectively capacitors of 50 \( \mu \)F, 70 \( \mu \)F, 90 \( \mu \)F and 110 \( \mu \)F in order to have an idea of its effect on the electromagnetic torque behavior inside the generator.

**4.1 Using a 50\( \mu \)F capacitor.**

![Figure 3 - Generator output voltage – Vab.](image-url)

![Figure 4 - Generator output voltage – Vbc.](image-url)

![Figure 5 - Generator output voltage – Vca.](image-url)

![Figure 6 - Generator output voltages - Vab, Vbc, Vca.](image-url)
4.2 Using a 70μF capacitor.

4.3 Using a 90μF capacitor.
Through figures 7, 12, and 17 can be clearly observed the presence of such oscillating electromagnetic torque, where the oscillations are always around the turbine nominal torque (7.5 N.m).

5 CONCLUSIONS

From the figures previously presented can be clearly observed the existence of heavy electromagnetic torque oscillations inside the generator; oscillations mainly due to the internal electromagnetic unbalance in the machine. This unbalance can be very damaging, in the mechanical point of view, to the generator performance, since they are the cause of highly oscillating rotational speed.

With the use of frequency domain modeling it is not possible to see clearly these effects. Therefore, it is evident that the mathematical model presented here, for a three-phase induction generator connected to a single-phase system, give us conditions and resources for the instantaneous analysis for the electromagnetic unbalance present inside the generator. In another words it is possible to make the analysis of generated voltage at the machine’s terminals. This model still give us conditions to evaluate a more adequate capacitor for the generator operating under such restrictions. So the mathematical model become a valuable tool for the design of such capacitor.

6 REFERENCES